6.6 Logarithmic and Exponential Equations

Learning Objectives

1. Solve Logarithmic Equations
2. Solve Exponential Equations
3. Solve Logarithmic and Exponential Equations Using a Graphing Utility
4. Solve Applications

Solving logarithmic equations

There are two types of logarithmic equations which use two different methods to solve.

1. If the equation has one logarithm, or it can be condensed to one logarithm, use the definition:
   - $y = \log_a x$ is equivalent to $x = a^y$ convert to exp. form

   $\log_B N = E \longrightarrow B^E = N$

2. If the equation has a single logarithm on both sides of the equation, use this property:
   - If $\log_a M = \log_a N$, then $M = N$ "drop" the logarithm from both sides

Example 1

Solving a Logarithmic Equation

Solve: $2\log_5 x = \log_5 9$

use Prop #5...
$r \cdot \log M = \log M^r$

$\log_5 x^2 = \log_5 9$

'drop' the logarithms

$x^2 = 9$

$x = 3$

($x \neq -3$ because domain of log. is > 0)

Example 2

Solving a Logarithmic Equation

Solve: $\underbrace{\log_5(x+6)+\log_5(x+2)}=1$

$\log_5\left[(x+6)(x+2)\right]=1 \leftarrow$ use Prop#3... $\log M + \log N = \log MN$

$\log_5(x^2+8x+12)=1$

$5^1 = x^2+8x+12$ $\leftarrow$ convert to exp. form

$0 = x^2+8x+7$

$0=(x+1)(x+7)$

$x=-1, \cancel{-7}$ .... you ~~must~~ check your solutions!

$x=-1$

Example 3

Solving a Logarithmic Equation

Solve: $\ln x = \underbrace{\ln(x+6) - \ln(x-4)}$

$$\ln x = \ln\left(\frac{x+6}{x-4}\right)$$ ← use Prop#4…
$\log M - \log N = \log \frac{M}{N}$

$$x = \frac{x+6}{x-4}$$ ← "drop" the logarithms

$$(x-4)\,x = \frac{x+6}{\cancel{x-4}}\,(\cancel{x-4})$$ ← mult. by the denominator

$$x^2 - 4x = x+6$$
$$x^2 - 5x - 6 = 0$$
$$(x-6)(x+1) = 0$$
$$x = 6, -1$$

check the solutions!

$x \neq -1$

$x = 6$

Your Turn

Solve $\underbrace{log_2(x-1) + log_2(x-3)}$ = 3

Condense to one log

$$\log_2\left[(x-1)(x-3)\right] = 3$$

$$\log_2(x^2-4x+3) = 3$$

convert to exp.

$$2^3 = x^2-4x+3$$

$$8 = x^2-4x+3$$

$$0 = x^2-4x-5$$

$$0 = (x-5)(x+1)$$

$x = 5, -1 \longrightarrow$ check! $x \neq -1$

$x = 5$

Solving Exponential Equations

There are different types of exponential equations which require different methods to solve.

1. If there is an exponential expression with the same base on both sides of the equation, use the property:

- If $a^u = a^v$ $then$ $u = v$.

if bases are equal, then the exponents are equal

2. If there is only one exponential expression in the equation, solve using the definition:

- $x = a^y$ is equivalent to $y = \log_a x$

$B^E = N \longrightarrow \log_B N = E$

convert to a logarit m

Pearson

Examples

Solve the exponential equations using the one-to-one property. (make both sides have same base)

(a) $5^{7-x} = 125 \longrightarrow 5^{7-x} = 5^3 \rightarrow 7 - x = 3$

$\boxed{x = 4}$

(b) $64^{2x} = 16^{x+1}$

$(4^3)^{2x} = (4^2)^{x+1} \rightarrow 4^{6x} = 4^{2x+2} \rightarrow 6x = 2x+2$

$4x = 2$

$\boxed{x = \frac{1}{2}}$

Example

Solve the exponential equations exactly and then approximate answers to four decimal places. (isolate the exponential)

$2(5^{3x}) + 3 = 19$

$-3 \quad -3$

$\dfrac{2(5^{3x})}{2} = \dfrac{16}{2}$

$5^{3x} = 8$

convert to log.

$\log_5 8 = 3x$

$\boxed{x = \dfrac{\log_5 8}{3} \text{ (exact)}}$

$\boxed{x \approx 0.4307 \text{ (approx.)}}$

Your Turn

Solve the exponential equation exactly and then approximate answer to four decimal places.

$$2(3^x) - 11 = 9$$

$$2(3^x) = 20$$

$$3^x = 10$$

$$\log_3 10 = x \text{ (exact)}$$

$$x \approx 2.0959 \text{ (approx)}$$

Example: Solving an Exponential Equation with Different Bases

Solve: $5^{x+3} = 3^{2x-5}$

Method #3... take the logarithm of both sides (can be any base)

$$\log\left(5^{x+3}\right) = \log\left(3^{2x-5}\right)$$

use Prop. #5... $\log M^r = r \cdot \log M$

$$(x+3)(\log 5) = (2x-5)(\log 3)$$

distribute

$$x \cdot (\log 5) + 3(\log 5) = 2x \cdot (\log 3) - 5(\log 3)$$

group the x terms

$$x \cdot (\log 5) - 2x(\log 3) = -3(\log 5) - 5(\log 3)$$

factor out x

$$x(\log 5 - 2\log 3) = -3\log 5 - 5\log 3$$

divide

$$x = \frac{-3\log 5 - 5\log 3}{\log 5 - 2\log 3} \approx 17.5597$$

(exact) (approx.)

Example - Solving an Exponential Equation That Is Quadratic in Form

$a=1$ $b=-1$ $c=-12$

Solve: $1(2^{2x})-1(2^x)-12=0$

(Method #4... factor and solve like a quadratic)

$$(2^x+3)(2^x-4)=0$$

$2^x+3=0$ OR $2^x-4=0$

$2^x \neq -3$ $\qquad$ $2^x=4$

$\log_2(-3)\neq x$ $\qquad$ $\log_2 4 = x$

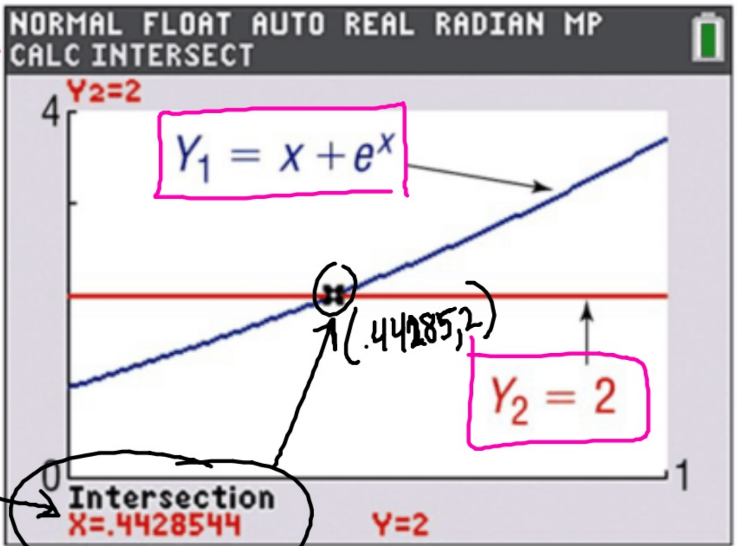
$\emptyset$ $\qquad$ $x=2$

Example: Solve $\underbrace{x + e^x}_{y_1} = \underbrace{2}_{y_2}$ graphically.

graph the two sides of the equation individually...

the intersection is the solution.

Use $\boxed{2^{nd}}$ $\boxed{\text{Trace}}$ Intersect on calculator to find the solution for x



$f(x) = C \cdot a^x$

Inital value (C), growth factor (a)

Example – Solve Application

The value *V* of a certain automobile that is *t* years old can be modeled by

$$V(t) = \overset{C}{14{,}467}(\overset{a}{0.8})^t$$

According to the model, when will the car be worth $9000?
(Round your answer to the nearest tenth.)

$$\frac{9000}{14467} = \frac{\cancel{14467}(0.8)^t}{\cancel{14467}}$$

$$0.6221... = 0.8^t$$

convert to log. form

$$\log_{0.8} 0.6221 = t$$

$$t \approx 2.1 \text{ years}$$

*Bonus Question: What is the rate (%) of decay per year?

The car is losing 20% of its value each year

$1 - 20\%$
$1 - .2$
0.8

growth by a %

$a = 1 + \%$ (grow)
$a = 1 - \%$ (decay)

Example – Solve Application

A population of bacteria cells is growing at a rate of 21% per hour.

$a = 1 + 21\%$

a) If there were 1000 cells in the original sample, write an exponential model for the population, P, of bacteria cells after t hours.

C

$P(t) = 1000(1.21)^t$

b) How many hours until there are 5000 bacteria cells?

$5000 = 1000(1.21)^t$

$5 = 1.21^t$

$\log_{1.21} 5 = t \rightarrow t \approx 8.4$ hours